Current status of ε_K and B meson spectrum test

Weonjong Lee (SWME)

Lattice Gauge Theory Research Center
Department of Physics and Astronomy
Seoul National University

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Contents

FNAL/MILC/SWME Collaboration
2012 — Present

FNAL/MILC/SWME Collaboration

- Seoul National University (SWME):
  Prof. Weonjong Lee
  Dr. Jon Bailey (RA Prof.),
  Dr. Nigel Cundy (RA Prof.)
  10+1 graduate students.
- Fermi National Accelerator Laboratory (FNAL):
  Prof. Andreas S. Kronfeld.
- University of Utah (MILC):
  Prof. Carleton Detar,
  Prof. Mehmet B. Oktay.

Lattice Gauge Theory Research Center (SNU)

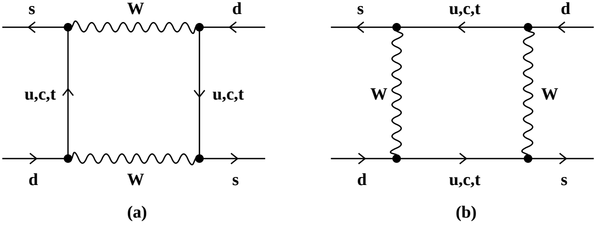
- Center Leader: Prof. Weonjong Lee.
- Research Assistant Prof.: Dr. Jon Bailey
- Research Assitant Prof.: Dr. Nigel Cundy
- 10+1 graduate students
- Secretary: Mrs. Sora Park.
- more details on `http://lgt.snu.ac.kr/`.

Group Photo (2014)

CP Violation and B_K

Kaon Eigenstates and ε

- Flavor eigenstates, $K^0 = (\bar{s}d)$ and $\bar{K}^0 = (s\bar{d})$ mix via box diagrams.



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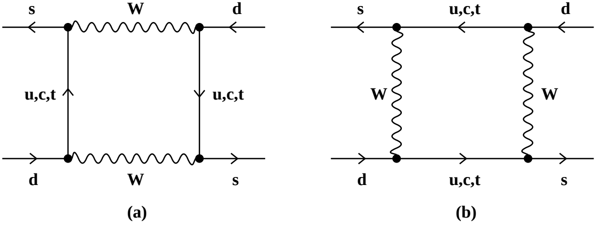

- CP eigenstates K_1(even) and K_2(odd).

$$K_1 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0) \qquad K_2 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0)$$

Kaon Eigenstates and ε

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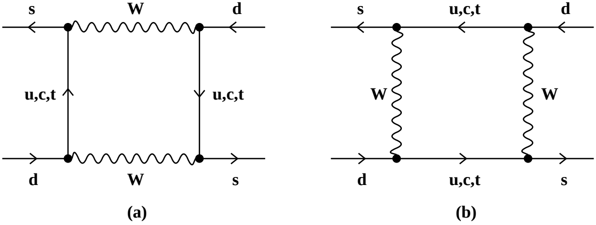


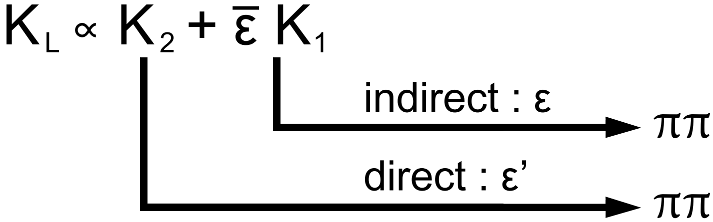
- CP eigenstates K_1(even) and K_2(odd).

$$K_1 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0) \qquad K_2 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0)$$

- Neutral Kaon eigenstates K_S and K_L.

$$K_S = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}(K_1 + \bar{\varepsilon} K_2) \qquad K_L = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}(K_2 + \bar{\varepsilon} K_1)$$

Indirect CP violation and direct CP violation



ε_K and $\hat{B}_K$, V_{cb} I

- Definition of ε_K

$$\varepsilon_K = \frac{A[K_L \to (\pi\pi)_{I=0}]}{A[K_S \to (\pi\pi)_{I=0}]}$$

- Relation between ε_K and $\hat{B}_K$ in standard model.

$$\varepsilon_K = \exp(i\phi_\varepsilon)\ \sin(\phi_\varepsilon)\ C_\varepsilon\ \mathrm{Im}\lambda_t\ X\ \hat{B}_K + \xi_0 + \xi_{LD}$$

$$X = \mathrm{Re}\lambda_c[\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t)] - \mathrm{Re}\lambda_t \eta_2 S_0(x_t)$$

$$\lambda_i = V_{is}^* V_{id}, \qquad x_i = m_i^2/M_W^2, \qquad C_\varepsilon = \frac{G_F^2 F_K^2 m_K M_W^2}{6\pi^2 \Delta M_K}$$

$$\xi_0 = \exp(i\phi_\varepsilon)\sin(\phi_\varepsilon)\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \approx 7\%$$

$\xi_{LD} =$ Long Distance Effect $\approx 2\%$ $\longrightarrow$ we neglect it!

ε_K and $\hat{B}_K$, V_{cb} II

- Inami-Lim functions:

$$S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3x_t^3 \ln(x_t)}{2(1-x_t)^3} \qquad \to +70.5\%$$

$$S_0(x_c, x_t) = x_c \left[\ln(\frac{x_t}{x_c}) - \frac{3x_t}{4(1-x_t)} - \frac{3x_t^2 \ln(x_t)}{4(1-x_t)^2} \right] \qquad \to +43.6\%$$

$$S_0(x_c) = x_c \qquad \to -14.1\%$$

- Dominant contribution (≈70.5%) comes with $|V_{cb}|^4$.

$$\mathrm{Im}\lambda_t \cdot \mathrm{Re}\lambda_t = \bar{\eta}\lambda^2 |V_{cb}|^4 (1-\bar{\rho})$$

$$\mathrm{Re}\lambda_c = -\lambda(1 - \frac{\lambda^2}{2}) + \mathcal{O}(\lambda^5)$$

$$\mathrm{Re}\lambda_t = -(1 - \frac{\lambda^2}{2}) A^2 \lambda^5 (1-\bar{\rho}) + \mathcal{O}(\lambda^7)$$

ε_K and $\hat{B}_K$, V_{cb} III

$$\mathrm{Im}\lambda_t = \eta A^2 \lambda^5 + \mathcal{O}(\lambda^7)$$

- Definition of B_K in standard model.

$$B_K = \frac{\langle \bar{K}_0 | [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d] | K_0 \rangle}{\frac{8}{3}\langle \bar{K}_0 | \bar{s}\gamma_\mu\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_\mu\gamma_5 d | K_0 \rangle}$$

$$\hat{B}_K = C(\mu) B_K(\mu), \qquad C(\mu) = \alpha_s(\mu)^{-\frac{\gamma_0}{2b_0}} [1 + \alpha_s(\mu) J_3]$$

- Experiment:

$$\varepsilon_K = (2.228 \pm 0.011) \times 10^{-3} \times e^{i\phi_\varepsilon}$$
$$\phi_\varepsilon = 43.52(5)^\circ$$

ε_K on the lattice

Wolfenstein Parameters

Input Parameters for Angle-Only-Fit (AOF)

- ϵ_K, $\hat{B}_K$, and $|V_{cb}|$ are used as inputs to determine the UT angles in the global fit of `UTfit` and `CKMfitter`.
- Instead, we can use angle-only-fit result for the UT apex $(\bar{\rho}, \bar{\eta})$.
- Then, we can take λ independently from

$$|V_{us}| = \lambda + \mathcal{O}(\lambda^7) \,,$$

  which comes from K_{l3} and $K_{\mu 2}$.
- Use $|V_{cb}|$ instead of A.

$$|V_{cb}| = A\lambda^2 + \mathcal{O}(\lambda^7)$$

λ	$0.22535(65)$	[1] CKMfitter
	$0.22535(65)$	[1] UTfit
	$0.2252(9)$	[1] $\lvert V_{us}\rvert$
$\bar{\rho}$	$0.131^{+0.026}_{-0.013}$	[1] CKMfitter
	$0.136(18)$	[1] UTfit
	$0.130(27)$	[2] UTfit(A)
$\bar{\eta}$	$0.345^{+0.013}_{-0.014}$	[1] CKMfitter
	$0.348(14)$	[1] UTfit
	$0.338(16)$	[2] UTfit(A)

Input Parameters of B_K, V_{cb} and others

B_K

$\hat{B}_K$	$0.7661(99)$	[3] FLAG
	$0.7379(47)(365)$	[4] SWME

V_{cb}

$V_{cb} \times 10^{-3}$	$42.42(86)$	[5] Incl.
	$39.04(49)(53)(19)$	[6] Excl.

Others

G_F	$1.1663787(6) \times 10^{-5}$ GeV^{-2}	[1]
M_W	$80.385(15)$ GeV	[1]
$m_c(m_c)$	$1.275(25)$ GeV	[1]
$m_t(m_t)$	$163.3(2.7)$ GeV	[7]
η_1	$1.43(23) \rightarrow 1.70(21)$	[8, 9]
η_2	$0.5765(65)$	[8]
η_3	$0.496(47)$	[10]
θ	$43.52(5)^\circ$	[1]
m_{K^0}	$497.614(24)$ MeV	[1]
ΔM_K	$3.484(6) \times 10^{-12}$ MeV	[1]
F_K	$156.1(8)$ MeV	[1]

ξ_0
Input Parameters

$$\xi_0 = \frac{\mathrm{Re}A_0}{\mathrm{Im}A_0}$$

ξ_0	$-1.63(19)(20) \times 10^{-4}$	[11]

- RBC-UKQCD collaboration performs lattice calculation of $\mathrm{Im}A_2$. From this result, ξ_0 can be obtained by the relation

$$\mathrm{Re}\left(\frac{\epsilon'_K}{\epsilon_K}\right) = \frac{1}{\sqrt{2}|\epsilon_K|}\omega\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \xi_0\right).$$

  Other inputs ω, ϵ_K and ϵ'_K/ϵ_K are taken from the experimental values.

- Here, we choose an approximation of $\cos(\phi_{\epsilon'} - \phi_\epsilon) \approx 1$.
- $\phi_\epsilon = 43.52(5)$, $\phi_{\epsilon'} = 42.3(1.5)$

ϵ_K: FLAG $\hat{B}_K$, AOF of $(\bar{\rho}, \bar{\eta})$, V_{us}

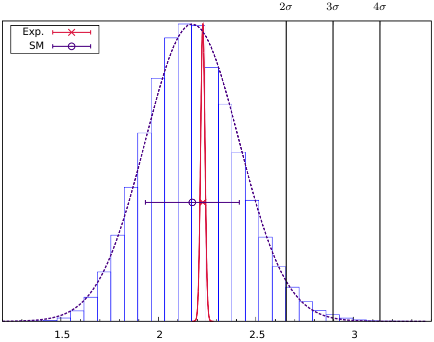


Figure: Inclusive V_{cb}

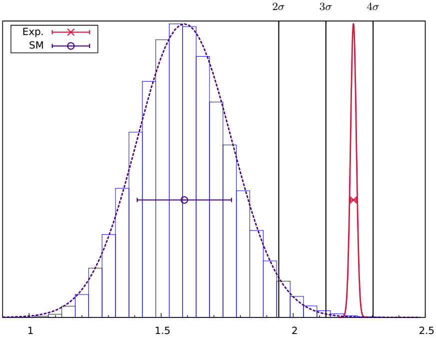


Figure: Exclusive V_{cb}

- With exclusive V_{cb}, it shows 3.6σ tension.

$$\epsilon_K^{Exp} = 2.228(11) \times 10^{-3}$$
$$\epsilon_K^{SM} = 1.588(178) \times 10^{-3}$$

ϵ_K: SWME $\hat{B}_K$, AOF of $(\bar{\rho}, \bar{\eta})$, V_{us}

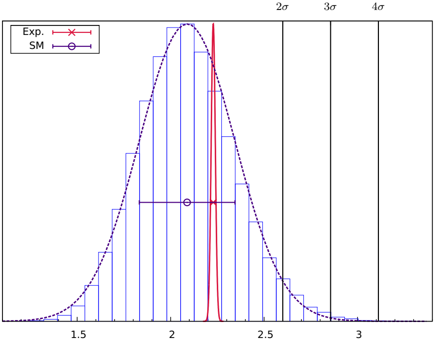


Figure: Inclusive V_{cb}

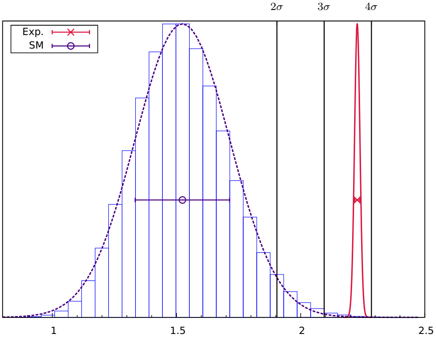


Figure: Exclusive V_{cb}

- With exclusive V_{cb}, it shows 3.7σ tension.

$$\epsilon_K^{Exp} = 2.228(11) \times 10^{-3}$$
$$\epsilon_K^{SM} = 1.524(190) \times 10^{-3}$$

Current Status of ε_K

- FLAG 2014: (in units of 1.0×10^{-3}, AOF)

$$\varepsilon_K = 1.59 \pm 0.18 \qquad \text{for Exclusive } V_{cb} \text{ (Lattice QCD)}$$
$$\varepsilon_K = 2.17 \pm 0.24 \qquad \text{for Inclusive } V_{cb} \text{ (QCD Sum Rule)}$$

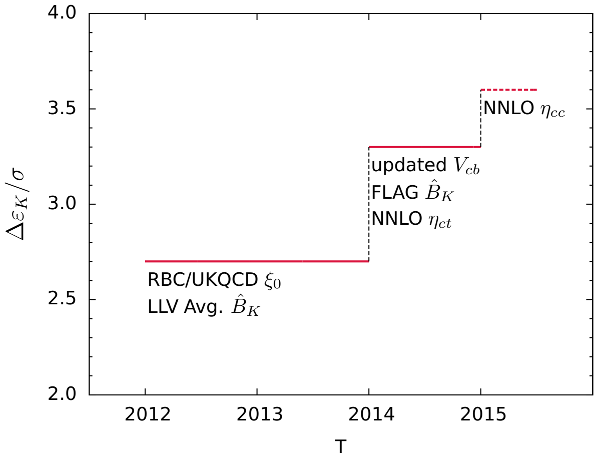
- Experiments:

$$\varepsilon_K = 2.228 \pm 0.011$$

- Hence, we observe 3.6(2) σ difference between the SM theory (Lattice QCD) and experiments.

- What does this mean? $\longrightarrow$ Breakdown of SM ?

Time Evolution of $\Delta\varepsilon_K$ on the Lattice



- $\Delta\varepsilon_K \equiv \varepsilon_K^{\text{exp}} - \varepsilon_K^{\text{SM}}$

Error Budget of Exclusive ε_K

source	error (%)	memo
V_{cb}	41.4	FNAL/MILC
$\bar{\eta}$	21.4	AOF
η_3	17.4	$c-t$ Box
$\bar{\rho}$	4.8	AOF
η_1	4.5	$c-c$ Box
m_t	3.6	
ξ_0	2.3	RBC/UKQCD
$\hat{B}_K$	1.6	FLAG
$\vdots$	$\vdots$	

V_{cb} on the lattice

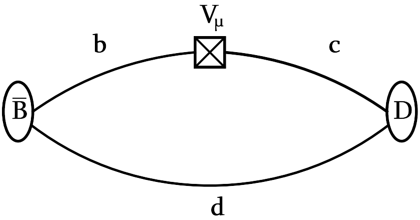
How to obtain V_{cb}

- Exclusive V_{cb} determination.
- $\bar{B} \to D + \ell + \bar{\nu}_\ell$
- $\bar{B} \to D^* + \ell + \bar{\nu}_\ell$

What to calculate on the lattice.

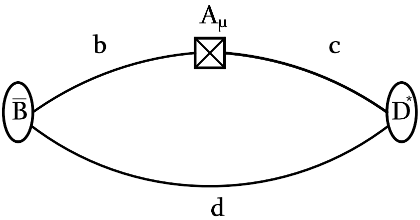
- $\langle D|Q_1|\bar{B}\rangle$ with $Q_1 = V_\mu,\ S$.

$$V_\mu = \bar{b}\gamma_\mu c$$
$$S = \bar{b}c$$



- $\langle D^*|Q_2|\bar{B}\rangle$ with $Q_2 = A_\mu,\ P$.

$$A_\mu = \bar{b}\gamma_\mu\gamma_5 c$$
$$P = \bar{b}\gamma_5 c$$



B_s meson mass

Motivation

- In heavy flavor physics, V_{cb} is of enormous interest.
- The dominant error in ϵ_K comes from V_{cb}.

$$\begin{cases} 41.4\% & \leftarrow V_{cb} \\ 1.6\% & \leftarrow \hat{B}_K \end{cases}$$

- $3.6(2)\sigma$ tension is observed using most up to date input parameters.

$$|\epsilon_K|^{\text{exp}} = 2.228(11) \times 10^{-3} \qquad \text{(PDG)}$$
$$|\epsilon_K|^{\text{SM}} = 1.588(178) \times 10^{-3} \qquad (\text{FLAG } \hat{B}_K, \text{FNAL/MILC } V_{cb})$$

- More precise determination of V_{cb} might lead to larger tension.
- Because the dominant error in V_{cb} comes from heavy quark discretization effect, we plan to use the OK action for the form factor calculation of the semi-leptonic decays

$$\bar{B} \to D^* l \nu_l\,, \qquad \bar{B} \to D l \nu_l.$$

- Here, we will verify the improvement in B meson spectrum.

OK Action (mass form)

$$S_{\rm OK} = S_{\rm Fermilab} + S_{\rm new}\,, \qquad S_{\rm Fermilab} = S_0 + S_B + S_E$$

$$\begin{aligned}
S_0 &= m_0 \sum_x \bar\psi(x)\psi(x) + \sum_x \bar\psi(x)\gamma_4 D_4 \psi(x) - \tfrac{1}{2}a \sum_x \bar\psi(x)\triangle_4\psi(x) \\
&\quad + \zeta \sum_x \bar\psi(x)\vec\gamma\cdot\vec D\psi(x) - \tfrac{1}{2} r_s \zeta a \sum_x \bar\psi(x)\triangle^{(3)}\psi(x) \\
&= \mathcal{O}(1) + \mathcal{O}(\lambda) \qquad [\lambda \sim a\Lambda,\ \Lambda/m_Q] \\
S_B &= -\tfrac{1}{2} c_B \zeta a \sum_x \bar\psi(x) i\vec\Sigma\cdot\vec B\psi(x) \to \mathcal{O}(\lambda) \\
S_E &= -\tfrac{1}{2} c_E \zeta a \sum_x \bar\psi(x)\vec\alpha\cdot\vec E\psi(x) \to \mathcal{O}(\lambda^2) \qquad (c_E \neq c_B :\ \text{OK action}) \\
m_0 &= \frac{1}{2\kappa_t} - (1 + 3 r_s \zeta + 18 c_4)
\end{aligned}$$

[M. B. Oktay and A. S. Kronfeld, PRD **78**, 014504 (2008)]

[A. El-Khadra, A. S. Kronfeld and P. B. Mackenzie, PRD **55**, 3933 (1997)]

OK Action (mass form)

$$
\begin{aligned}
S_{\text{new}} = \mathcal{O}(\lambda^3) &= c_1 a^2 \sum_x \bar{\psi}(x) \sum_i \gamma_i D_i \triangle_i \psi(x) \\
&+ c_2 a^2 \sum_x \bar{\psi}(x) \{\vec{\gamma} \cdot \vec{D}, \triangle^{(3)}\} \psi(x) \\
&+ c_3 a^2 \sum_x \bar{\psi}(x) \{\vec{\gamma} \cdot \vec{D}, i\vec{\Sigma} \cdot \vec{B}\} \psi(x) \\
&+ c_{EE} a^2 \sum_x \bar{\psi}(x) \{\gamma_4 D_4, \vec{\alpha} \cdot \vec{E}\} \psi(x) \\
&+ c_4 a^3 \sum_x \bar{\psi}(x) \sum_i \triangle_i{}^2 \psi(x) \\
&+ c_5 a^3 \sum_x \bar{\psi}(x) \sum_i \sum_{j \neq i} \{\mathrm{i}\Sigma_i B_i, \triangle_j\} \psi(x)
\end{aligned}
$$

OK Action: Tadpole Improvement (hopping form)

$$
\begin{aligned}
& c_5 a^3 \bar{\psi}(x) \sum_i \sum_{j \neq i} \{ \mathrm{i}\Sigma_i B_{i\text{lat}}, \triangle_{j\text{lat}} \} \psi(x) \\
& = \mathrm{i}\frac{2\tilde{c}_5\tilde{\kappa}_t}{4u_0^2} \bar{\psi}_x \sum_i \Sigma_i T_i^{(3)} \psi_x - \mathrm{i}\frac{32\tilde{c}_5\tilde{\kappa}_t}{2u_0^3} \bar{\psi}_x \overrightarrow{\Sigma} \cdot \overrightarrow{B} \psi_x \\
& + \mathrm{i}\frac{2\tilde{c}_5\tilde{\kappa}_t}{u_0^4} \bar{\psi}_x \sum_i \Big(-\frac{1}{4}\Sigma_i T_i^{(3)} + \sum_{j \neq i} \{ \Sigma_i B_i, (T_j + T_{-j}) \} \Big) \psi_x
\end{aligned}
$$

$$
T_i^{(3)} \equiv \sum_{j,k=1}^{3} \epsilon_{ijk} \Big(T_{-k}(T_j - T_{-j})T_k - T_k(T_j - T_{-j})T_{-k} \Big)
$$

Measurement

Gauge Ensemble, Heavy Quark κ, Meson Momentum

- MILC asqtad $N_f = 2+1$

a(fm)	$N_L^3 \times N_T$	β	am'_l	am'_s	u_0	a^{-1}(GeV)	N_{conf}	$N_{t_{\text{src}}}$
0.12	$20^3 \times 64$	6.79	0.02	0.05	0.8688	1.683^{+43}_{-16}	484	6
0.15	$16^3 \times 48$	6.60	0.029	0.0484	0.8614	1.350^{+35}_{-13}	500	4

- Meson mass at $a \cong 0.12$ fm

$\tilde{\kappa}$	0.038	0.039	0.040	0.041
$aM(B_s)$	3.99	3.65	3.32	3.01
$aM(\eta_b)$	6.75	6.17	5.61	5.06

- 11 momenta $|\boldsymbol{p}a| = 0, 0.099, \cdots, 1.26$

Measurement: Interpolating Operator

- Meson correlator

$$C(t,\boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{x}} \langle \mathcal{O}^\dagger(t,\boldsymbol{x})\mathcal{O}(0,\boldsymbol{0})\rangle$$

- Heavy-light meson interpolating operator

$$\mathcal{O}_{\mathsf{t}}(x) = \bar{\psi}_\alpha(x)\Gamma_{\alpha\beta}\Omega_{\beta\mathsf{t}}(x)\chi(x)$$

$$\Gamma = \begin{cases} \gamma_5 & \text{(Pseudo-scalar)} \\ \gamma_\mu & \text{(Vector)} \end{cases} , \quad \Omega(x) \equiv \gamma_1^{x_1}\gamma_2^{x_2}\gamma_3^{x_3}\gamma_4^{x_4}$$

- Quarkonium interpolating operator

$$\mathcal{O}(x) = \bar{\psi}_\alpha(x)\Gamma_{\alpha\beta}\psi_\beta(x)$$

[Wingate *et al.*, PRD **67**, 054505 (2003) , C. Bernard *et al.*, PRD **83**, 034503 (2011)]

Measurement: Interpolating Operator

Smearing

- For heavy quark, we also used a smeared sink using the Richardson 1S charmonium wave function $S(\boldsymbol{x})$.

$$\phi(t, \boldsymbol{x}) = \sum_{\boldsymbol{y}} S(\boldsymbol{y}) \psi(t, \boldsymbol{x} + \boldsymbol{y}) \,.$$

- For a smeared correlator, we applied the Coulomb gauge fixing.
- Analysis for smeared correlators is not done. So, we will present the results for the point source and point sink data.

[C. Bernard *et al.*, PRD **83**, 034503 (2011)]

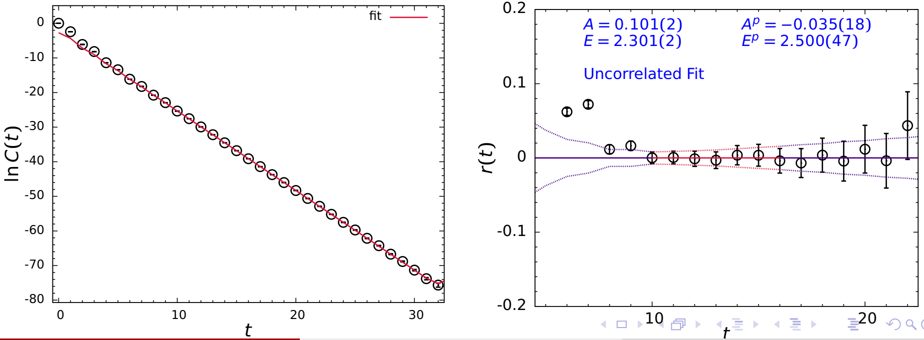
Correlator Fit

- fit function

$$f(t) = A\{e^{-Et} + e^{-E(T-t)}\} + (-1)^t A^p\{e^{-E^p t} + e^{-E^p(T-t)}\}$$

- fit residual

$$r(t) = \frac{C(t) - f(t)}{|C(t)|}, \text{where } C(t) \text{ is data.}$$



Correlator Fit: Effective Mass

$$m_{\text{eff}}(t) = \frac{1}{2}\ln\left(\frac{C(t)}{C(t+2)}\right)$$

For small t,

$$\begin{aligned} C(t) \cong& A(e^{-Et} + \beta e^{-(E+\Delta E)t}) \\ =& Ae^{-Et}(1 + \beta e^{-(\Delta E)t}) \ , \end{aligned}$$

$$\begin{cases} \beta > 0 & \text{(excited state)} \\ \beta \sim -(-1)^t & \text{(time parity state)} \end{cases}$$

$$m_{\text{eff}} \approx E + \beta(\Delta E)e^{-(\Delta E)t}$$

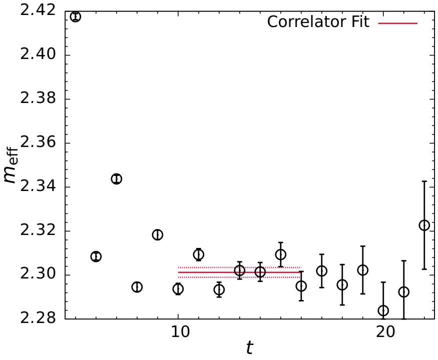


Figure: $[\overline{Q}q$, PS, $\tilde{\kappa} = 0.038$, $\boldsymbol{p} = \boldsymbol{0}]$

Dispersion Relation

$$E = M_1 + \frac{\boldsymbol{p}^2}{2M_2} - \frac{(\boldsymbol{p}^2)^2}{8M_4^3} - \frac{a^3 W_4}{6}\sum_i p_i^4$$

$$\widetilde{E} = E + \frac{a^3 W_4}{6}\sum_i p_i^4 \,, \quad \boldsymbol{n} = (2,2,1)\,, (3,0,0)$$

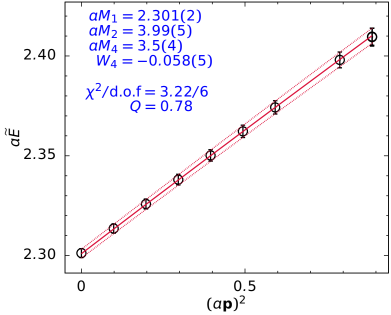


Figure: $[\overline{Q}q$, PS, $\tilde{\kappa} = 0.038]$

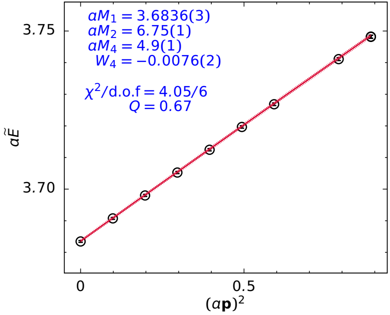


Figure: $[\overline{Q}Q$, PS, $\tilde{\kappa} = 0.038]$

Improvement Test: Inconsistency Parameter

$$I \equiv \frac{2\delta M_{\overline{Q}q} - (\delta M_{\overline{Q}Q} + \delta M_{\overline{q}q})}{2M_{2\overline{Q}q}} = \frac{2\delta B_{\overline{Q}q} - (\delta B_{\overline{Q}Q} + \delta B_{\overline{q}q})}{2M_{2\overline{Q}q}}$$

$$M_{1\overline{Q}q} = m_{1\overline{Q}} + m_{1q} + B_{1\overline{Q}q} \qquad \delta M_{\overline{Q}q} = M_{2\overline{Q}q} - M_{1\overline{Q}q}$$
$$M_{2\overline{Q}q} = m_{2\overline{Q}} + m_{2q} + B_{2\overline{Q}q} \qquad \delta B_{\overline{Q}q} = B_{2\overline{Q}q} - B_{1\overline{Q}q}$$

[S. Collins *et al.*, NPB **47**, 455 (1996) , A. S. Kronfeld, NPB **53**, 401 (1997)]

- Inconsistency parameter I can be used to examine the improvements by $\mathcal{O}(\boldsymbol{p}^4)$ terms in the action. OK action is designed to improve these terms and matched at tree-level.
- Binding energies B_1 and B_2 are of order $\mathcal{O}(\boldsymbol{p}^2)$. Because the kinetic meson mass M_2 appears with a factor $\boldsymbol{p}^2$, the leading contribution of binding energy B_2 generated by $\mathcal{O}(\boldsymbol{p}^4)$ terms in the action.

$$E = M_1 + \frac{\boldsymbol{p}^2}{2M_2} + \cdots = M_1 + \frac{\boldsymbol{p}^2}{2(m_{2\overline{Q}} + m_{2q})}\left[1 - \frac{B_{2\overline{Q}q}}{(m_{2\overline{Q}} + m_{2q})} + \cdots\right] + \cdots$$

Improvement Test: Inconsistency Parameter

$$I \cong \frac{2\delta M_{\overline{Q}q} - \delta M_{\overline{Q}Q}}{2M_{2\overline{Q}q}} \cong \frac{2\delta B_{\overline{Q}q} - \delta B_{\overline{Q}Q}}{2M_{2\overline{Q}q}}$$

- Considering non-relativistic limit of quark and anti-quark system, for S-wave case ($\mu_2^{-1} = m_{2\overline{Q}}^{-1} + m_{2q}^{-1}$),

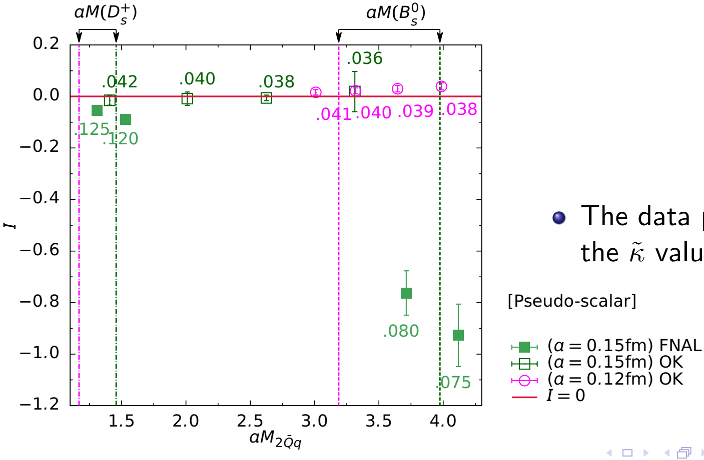
$$\begin{aligned} \delta B_{\overline{Q}q} \quad &= \frac{5}{3}\frac{\langle \boldsymbol{p}^2 \rangle}{2\mu_2}\Big[\mu_2\Big(\frac{m_{2\overline{Q}}^2}{m_{4\overline{Q}}^3} + \frac{m_{2q}^2}{m_{4q}^3}\Big) - 1\Big] \qquad (m_4 : c_1, c_3) \\ &+ \frac{4}{3}a^3\frac{\langle \boldsymbol{p}^2 \rangle}{2\mu_2}\mu_2(w_{4\overline{Q}} m_{2\overline{Q}}^2 + w_{4q} m_{2q}^2) \quad (w_4 : c_2, c_4) \\ &+ \mathcal{O}(p^4) \end{aligned}$$

[A. S. Kronfeld, NPB **53**, 401 (1997) , C. Bernard *et al.*, PRD **83**, 034503 (2011)]

- Leading contribution of $\mathcal{O}(\boldsymbol{p}^2)$ in δB vanishes when $w_4 = 0$, $m_2 = m_4$, not only for S-wave states but also for higher harmonics.

Improvement Test: Inconsistency Parameter

- The coarse $(a = 0.12\text{fm})$ ensemble data covers the B_s^0 mass and shows significant improvement compared to the Fermilab action.



- The data point labels denote the $\tilde{\kappa}$ values.

Improvement Test: Hyperfine Splitting Δ

$$\Delta_1 = M_1^* - M_1\,,\ \Delta_2 = M_2^* - M_2$$

Recall,

$$\begin{aligned} M^{(*)}_{1\overline{Q}q} &= m_{1\overline{Q}} + m_{1q} + B^{(*)}_{1\overline{Q}q} \\ M^{(*)}_{2\overline{Q}q} &= m_{2\overline{Q}} + m_{2q} + B^{(*)}_{2\overline{Q}q} \\ \delta B^{(*)} &= B_2^{(*)} - B_1^{(*)} \end{aligned}$$

Then,

$$\Delta_2 = \Delta_1 + \delta B^* - \delta B$$

- The difference in hyperfine splittings $\Delta_2 - \Delta_1$ also can be used to examine the improvement from $\mathcal{O}(\boldsymbol{p}^4)$ terms in the action.

Improvement Test: Hyperfine Splitting Δ

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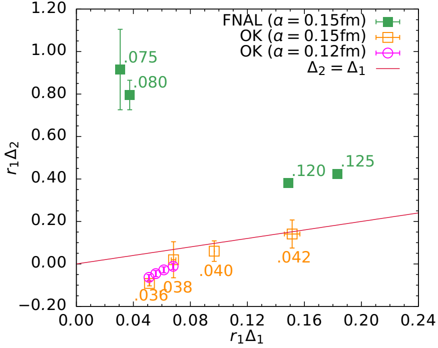

Figure: Quarkonium

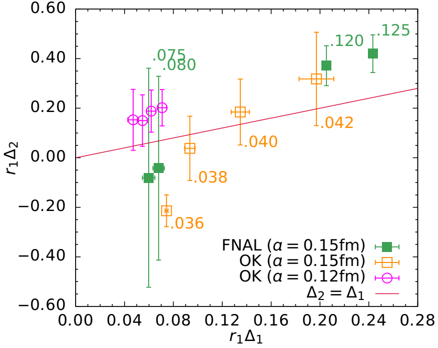

Figure: Heavy-light

Conclusion and Outlook

- Inconsistency parameter shows that the OK action clearly improves $\mathcal{O}(\boldsymbol{p}^4)$ terms.
- Hyperfine splitting shows that the OK action clearly improves the higher dimension magnetic effects for the quarkonium.
- For heavy-light system, errors of hyperfine splittings on 0.15fm data are too large to draw any conclusion.
- We plan to calculate V_{cb} with the highest precision possible.
- Improved current relevant to the decay $\bar{B} \to D^* l\nu$ at zero recoil is needed. (Jon A. Bailey and J. Leem)
- We plan to calculate the 1-loop coefficients for c_B and c_E in the OK action. (Y.C. Jang)
- Highly optimized inverter using QUDA will be available soon. (Y.C. Jang)

Grand Challenges in the front

Tentative Goals (1)

1. We would like to determine B_K directly from the standard model with its systematic and statistical error $\leq 2\%$.

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4. In addition, we need to obtain the matching factor using NPR (Jangho Kim) and using the two-loop perturbation theory $(\cdots)$.
   ※ matching error $< 1.0\%$

Tentative Goals (2)

1. V_{cb}, we need to calculate the following semi-leptonic form factors:

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4. Our goal is to determine V_{cb} with its statistical and systematic error $\leq 0.5\%$.

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2. Here, the precision goal is only $10 \sim 15\%$.
3. We need $N_f = 2 + 1 + 1$ calculation on the lattice. MILC provides HISQ ensembles with $N_f = 2 + 1 + 1$.
4. As a by-product, a substantial gain is that the charm quark mass dependence might be under control in this way. (Brod and Gorbahn)

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2. As a result, we would like to provide a crucial clue to the physics beyond the standard model.
3. As a result, we would like to guide the whole particle physics community into a new world beyond the standard model.

Thank God for your help !!!

References for the Input Parameters I

[1] J. Beringer et al.
Review of Particle Physics (RPP).
Phys.Rev., D86:010001, 2012.

[2] A. Bevan, M. Bona, M. Ciuchini, D. Derkach, E. Franco, et al.
Standard Model updates and new physics analysis with the Unitarity Triangle fit.
Nucl.Phys.Proc.Suppl., 241-242:89–94, 2013.

[3] Sinya Aoki, Yasumichi Aoki, Claude Bernard, Tom Blum, Gilberto Colangelo, et al.
Review of lattice results concerning low energy particle physics.
2013.

[4] Taegil Bae et al.
Improved determination of B_K with staggered quarks.
Phys.Rev., D89:074504, 2014.

[5] Paolo Gambino and Christoph Schwanda.
Inclusive semileptonic fits, heavy quark masses, and V_{cb}.
Phys.Rev., D89:014022, 2014.

References for the Input Parameters II

[6] Jon A. Bailey, A. Bazavov, C. Bernard, C.M. Bouchard, C. DeTar, et al.
Update of $|V_{cb}|$ from the $\bar{B} \to D^* \ell \bar{\nu}$ form factor at zero recoil with three-flavor lattice QCD.
2014.

[7] S. Alekhin, A. Djouadi, and S. Moch.
The top quark and Higgs boson masses and the stability of the electroweak vacuum.
Phys.Lett., B716:214–219, 2012.

[8] Andrzej J. Buras and Diego Guadagnoli.
Phys.Rev., D78:033005, 2008.

[9] Andrzej J. Buras and Jennifer Girrbach.
Stringent tests of constrained Minimal Flavor Violation through ΔF =2 transitions.
Eur.Phys.J., C73(9):2560, 2013.

[10] Joachim Brod and Martin Gorbahn.
ϵ_K at Next-to-Next-to-Leading Order: The Charm-Top-Quark Contribution.
Phys.Rev., D82:094026, 2010.

[11] T. Blum, P.A. Boyle, N.H. Christ, N. Garron, E. Goode, et al.
Phys.Rev.Lett., 108:141601, 2012.